Market Imperfections and Crowdfunding*

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Abstract

This paper offers a model of crowdfunding that represents a growing area of interest among practitioners and theorists. It is one of the first articles analyzing the choice between the different types of crowdfunding (reward-based vs. equity-based) and the choice between crowdfunding and traditional financing. The model is based on standard market imperfections such as asymmetric information and moral hazard as well as on some specific features of crowdfunding including the market feedback regarding new projects. The model provides several implications, most of which have not yet been tested. For example, we find that when asymmetric information is important, high-quality projects prefer reward-based crowdfunding. The choice of an all-or-nothing mechanism as opposed to a keep-it-all can serve as a signal of a firm's quality ("signalling by risk-bearing"). Crowdfunding is selected over a traditional bank loan if the demand for the product is either very small or very large.

Keywords: crowdfunding, asymmetric information, moral hazard, equity-based crowdfunding, reward-based crowdfunding

JEL Codes: D82, G32, L11, L26, M13

1 Introduction

Crowdfunding is the practice of funding a start-up company or project by raising funds from a large number of people. It is usually performed on-line. The volume of funds raised using crowdfunding has been quickly growing the last 5-7 years. In 2009 the volume of funds raised using crowdfunding was negligeably small.

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In 2014, crowdfunding platforms raised $16.2 billion, which is an increase from the $6.1 billion raised in 2013. Crowdfunding has more than doubled since then and raised $34.4 billion in 2015. Some analysts predict that crowdfunding market size will grow at an annual rate of 27.8% and will surpass venture capital investments in the near future.[1] Kickstarter, which is the leading crowdfunding platform in the US, has raised over $2.4 billion in pledges from 10.9 million backers to fund almost 107,000 creative ideas.[2]

Crowdfunding research is quickly growing.[3] As we are writing this article, the number of empirical papers significantly exceeds the number of theoretical papers. Empirical papers on crowdfunding have found the following: crowdfunding relaxes geographic constraints on fundraising, which inhibit venture capital and angel financing (Agrawal et al. (2010)); asymmetric information and signalling seem to play a significant role in crowdfunding (Ahlers, Cumming, Guenther, and Schweizer (2015), Hildebrand, Puri, and Rocholl (2014)); success of a project and its delays are related to the volume of financing it receives (Mollick (2014)); the timing of contributions usually follows a pattern (Kuppuswamy and Bayus (2015b)). Yet, the literature still lacks a full understanding of how entrepreneurs choose between different types of crowdfunding and how they decide whether to use crowdfunding or other types of financing. In this paper we try to shed some light on these questions. We build a model that addresses some of the aspects of crowdfunding mentioned above. In addition, our model incorporates other major features of crowdfunding. For example, in the case of crowdfunding the market provides intense feedback regarding a firm's projects and products. Unlike venture capital and bank financing, there is no major investor with crowdfunding, who often maintain a certain degree of monitoring and control over the firm's activities. As a result, the entrepreneur may be subject to a higher dregree of moral hazard (Agrawal, Catalini, and Goldfarb (2013), Moritz and Block (2014), Strausz (2017)). Finally, the owners may have better information about the quality of their products and their costs.

We focus on the two leading types of crowdfunding: reward-based crowdfunding (used by Kickstarter-the leading platform in the area) and equity-based crowdfunding. In the case of reward-based crowdfunding, investors count on some extra-benefits from the company such as future product discounts. Under equity-based crowdfunding investors will receive shares of the company. Reward-based crowdfunding campaigns are commonly offered in one of two models. The "Keep-It-All" (KIA) model involves the entrepreneurial firm setting a fundraising goal and keeping the entire amount raised, regardless of whether or not they meet their goal, thereby allocating the risk to the crowd when an underfunded project goes ahead. The "All-Or-Nothing" (AON) model involves the entrepre-

[1] See, for example, Salman (2016) or a global crowdfunding report on http://crowdfundbeat.com/2016/02/03/report-global-crowdfunding-market-2016-2020/

[2] Kickstarter website (June 1, 2016):
https://www.kickstarter.com/help/stats?ref=about_subnav

[3] Moritz and Block (2014) and Kuppuswamy and Bayus (2015a) provide a review of the literature in this field. For international aspects of crowdfunding see, for example, Gabison (2015), Miglo (2017), or Hatfield (2017).

neurial firm setting a fundraising goal and keeping nothing unless the goal is achieved, thereby shifting the risk to the entrepreneur. Kickstarter follows an "all or nothing" or threshold model, so funders' pledged money is only collected if the goal is reached. While other crowdfunding efforts do not always follow this model, it is currently the dominant approach to crowdfunding, and parallels the way that other funding efforts for new ventures work. Our model is also reflective of the fact that crowdfunding is an area where production decisions and finance are closely connected. The crowdfunding method choice directly and indirectly affects the development of a project and its promotion, production scale and price decisions.

In our model each type of financing has its cost and benefits. Under reward-based crowdfunding, it is harder to achieve the fundraising goal with large projects since the funders' potential benefits do not include the firm's long-term profits unlike under equity-based crowdfunding. On the other hand, the entrepreneur's stake of equity is reduced under equity-based crowdfunding, which affects pricing and production decisions. In particular, we find that in this case, prices are higher and production quantities are lower than optimal since the entrepreneur receives less than 100% of the benefits from increasing production while bears a non-shared extra-cost, therefore, the entrepreneur chooses a lower level of production. We also find that high-quality projects are likely to chose reward-based crowdfunding as a signal of quality. Also, they are more likely to be funded through the AON scheme. Low-quality or high-risk projects are less likely to mimick high-quality firms and chose AON, which implies more fundrasing responsability and risks, and prefer KIA instead. Traditional bank financing may lead to bankruptcy if the firm is unsuccessful. So the magnitude of the bankruptcy cost plays a role in the financing method choice. If these costs are high enough, the entrepreneur may prefer crowdfunding since, formally, crowdfunding does not neccessarily lead to bankruptcy if the crowdfunding campaign or production fails. However, under reward-based crowdfunding, indirect costs of distress may arise related to consumer protection law in case products are not delivered to customers. We find that a separating equilibrium where high-quality firms select reward-based crowdfunging can only exist if these costs are relatively high. Finally, unlike traditional financing, crowdfunfing provides market feedback. When this feature of crowdfunding is introduced into the basic model, we find that crowdfunding is selected over a traditional bank loan if the demand for the product is either very small or very large.

As was mentioned previously, the number of theoretical papers on crowdfunding is relatively small. Note the following. Belleflamme, Lambert, and Schwienbacher (2010) identify a number of issues related to crowdfunding from an industrial organization perspective. In their model, they analyze reward-based crowdfunding with pre-ordering and price discrimination, and study the conditions under which crowdfunding is preferred to traditional forms of external funding. In the second model, crowdfunding is a way to make a product better known to consumers. The authors argue that non-profit organizations tend to be more successful in using crowdfunding.

Belleammey, Lambertz and Schwienbacher (2014) compare reward-based

and equity-based crowdfunding. In either case, the funders enjoy community benefits that increase their utility. It is shown that the entrepreneur prefers pre-ordering if the initial capital requirement is relatively small compared to the market size and prefers profit sharing otherwise. Belleammey et al (2014) also offer some extensions on the impact of quality uncertainty and information asymmetry but in these extensions the choice between different forms of crowdfunding and other forms of financing is not modelled. As the authors mentioned, further research is required.

Strausz (2017) studies entrepreneurs' interactions with customers before investment using the mechanism design approach. Under aggregate demand uncertainty, crowdfunding improves the screening of potential customers. Entrepreneurial moral hazard threatens this benefit. Studying the subsequent tradeoff between screening and moral hazard, the paper characterizes optimal mechanisms. Efficiency is sustainable only if returns exceed investment costs by a margin reflecting the degree of moral hazard. Constrained efficient mechanisms exhibit underinvestment.

Hu, Li, and Shi (2014) study the optimal product and pricing decisions in a crowdfunding all-or-nothing mechanism. When the buyers are sufficiently heterogenous in their product valuations, the creator should offer a line of products with different levels of product quality. Compared to the traditional situation where orders are placed and fulfilled individually, with the crowdfunding mechanism, a product line is more likely to be optimal than a single product and the quality gap between products is smaller. The paper also shows the effect of the crowdfunding mechanism on pricing dynamics over time. Together, these results underscore the substantial influence of the emerging crowdfunding mechanisms on common marketing decisions.

The rest of the paper is organized as follows. Section 2 describes the basic model and some preliminary results. Section 3 through 6 discuss the consequences of introducing different kinds of market imperfections into the basic model and their implications for crowdfunding decisions. Section 7 analyzes cases that involve several market imperfections simultaneously. Section 8 discusses the consistency of the model's predictions with observed empirical evidence. Section 9 discusses the model's robustness and its potential extensions and Section 10 is a conclusion to the study.

2 Basic Model

An entrepreneurial firm has monopoly power over its innovative product or service. The fixed costs of launching the production equal I. The firm intends to sell its product in two consecutive periods. In period $t = 1, 2$, if the firm produces q_t units, it costs cq_t in total. The demand for the good in each period is given by the inverse demand function $p_t = a - q_t$.[4] The firm needs funds to

[4] Some papers use the approach where there are individual customers with different demand functions (see, for example, Belleammey et al (2014) and Hu, Li and Shi (2014)) or where there is a possibility of product substitution between periods. Section 9 discusses the model's

cover its start-up costs and is considering crowdfunding. Under reward-based crowdfunding the firm collects pre-orders for period 1.[5] Under equity-based crowdfunding, the firm sells a fraction α of the firm. Funders and entrepreneurs are assumed to be risk-neutral and the risk-free interest rate is 0. A two-period model will help us understand the difference between basic features of different types of crowdfunding (like long-term character of earnings in the case of equity-based crowdfunding vs. short-term rewards in the case of reward-based crowdfunding) as well as capture some other important features of crowdfunding such as incorporating market feedback during period 1 into the product quality. Since crowdfunding is usually used to cover the start-up costs, period 2 financing is not explicitly modelled. The capital structure and the ownership structure will remain the same in period 2 as they are at the end of period 1. Earnings will be distributed accordingly.

2.1 Reward-Based Crowdfunding: Pre-orders

The timing of events is as follows:

1. Firm selects p_1 (pre-order price). The demand for the product is determined. If $p_1 q_1 < I + cq_1$, the firm is liquidated.[6] Otherwise, the entrepreneur collects profit $(p_1 - c)(a - p_1) - I$.

2. Firm selects p_2. The entrepreneur collects profit $(p_2 - c)(a - p_2)$.

In this setting, the firm selects a pre-order price in order to maximize its profits. The constraint, however, comes from the necessity to collect the amount of money required to launch production.

In period 2, the firm chooses p_2 to maximize $(p_2 - c)(a - p_2)$, which gives $p_2 = \frac{a+c}{2}$.

In period 1, the firm maximizes $(p_1 - c)(a - p_1) - I$ subject to: $p_1 q_1 = p_1(a - p_1) \geq I + cq_1 = I + c(a - p_1)$. This condition means that the amount of pre-orders should cover the start-up cost (fixed costs and the period 1's variable costs).

Two cases are possible. If

$$\frac{(a-c)^2}{4} \geq I \tag{1}$$

robustness with regard to changes in the demand functions and other features of the model.

[5] Existing studies consider consumer financing through pre-ordering, bootstrap financing (see, e.g., Winborg and Landstrom, 2001; and Ebben and Johnson, 2006) or working capital loans. However, they do not usually distinguish between advance payments made at the very beginning of an entrepreneurial initiative and those made during the course of further developments. Crowdfunding pertains specifically to the financing of innovative entrepreneurial projects. There are many features of this type of financing such as market feedback from a large number of funders, which is typically not included in existing studies on consumer financing or bootstrap financing etc.

[6] The presence of thresholds for the minimum required amount of funds (the poject fails if the thresholds are not met) is typical in crowdfunding. We begin our analysis with a natural assumption as to why this occurs: a firm's inability to cover the start-up costs. In Section 5 we will discuss other reasons for possible thresholds.

then $p_1 = \frac{a+c}{2}$.

The firm's profit over the two periods equals

$$\Pi = \frac{(a-c)^2}{4} - I + \frac{(a-c)^2}{4} = \frac{(a-c)^2}{2} - I \tag{2}$$

If (1) fails, the firm will not be able to raise the funds needed to launch the production. When the required amount of initial investment is quite large, reward-based crowdfunding may not be an option.

2.2 Equity-Crowdfunding: Profit-Sharing

The timing of events is as follows:

1. Firm selects α (the fraction of the firm for sale) and p_1 and sells α for price M. If $M < I + cq_1$, the firm is liquidated.

2. Firm selects p_2.

In this setting, the firm has more flexibility in rasing the initial amount of investments, since the funders can also count on the second period's (future) profit.

In period 2, the firm chooses p_2 to maximize the entrepreneur‘s profit $(1-\alpha)(p_2-c)(a-p_2)$, which makes $p_2 = \frac{a+c}{2}$. The firm's profit in period 2 is $\frac{(a-c)^2}{4}$.

In period 1, the firm chooses α and p_1 to maximize the entreprneur's expected profit over the two periods:

$$(1-\alpha)(p_1(a-p_1) + M - I - cq_1 + \frac{(a-c)^2}{4}) \tag{3}$$

subject to

$$M \geq I + cq_1 \tag{4}$$

The funders‘ expected earnings should cover their investment cost or:

$$\alpha(p_1(a-p_1) + \frac{(a-c)^2}{4}) \geq M \tag{5}$$

For the optimal solution the conditions (4) and (5) will be binded because the firm can always make α as small as necessary to satisfy them. Then we have:

$$\alpha = \frac{I + cq_1}{p_1(a-p_1) + \frac{(a-c)^2}{4}} \tag{6}$$

Substituting this into (3) makes the entrepreneur‘s expected profit over the two periods equal to:

$$(p_1 - c)(a - p_1) - I + \frac{(a-c)^2}{4}$$

This implies $p_1 = \frac{a+c}{2}$.

The entrepreneur's expected profit then equals

$$\frac{(a-c)^2}{4} - I + \frac{(a-c)^2}{4} = \frac{(a-c)^2}{2} - I \tag{7}$$

As we can see, it is the same amount as in (2). This is not surprising given that in the absence of any financial market imperfections every type of financing should have the same result (similar to Modigliani-Miller proposition (1958)) as long as they fit into the budget constraints.

Lemma 1. *If I is sufficiently small ($\frac{(a-c)^2}{4} \geq I$), the firm is indifferent between reward-based and equity-based crowdfunding. If I is large, equity-based crowdfunding is preferred.*

The proof of this lemma follows from the above analysis. If I is small, the firm's profit is the same under the two types of crowdfunding ((2) and (7)). If I is large, it follows from the previous subsection that the firm is not able to raise enough funds to cover its start-up costs using a reward-based crowdfunding.

Lemma 1 shows that equity-based crowdfunding has a "technical" advantage for large projects (high fixed costs I and high variable costs c). Since our focus is on the role of market imperfections, we will usually assume that condition (1) holds in the further analysis, i.e. both types of crowdfunding are feasible.

3 Moral hazard: costly entrepreneurial effort

So far we assumed that the decisions about α and p_1 are made simultaneously. We know, however, that under equity-based crowdfunding, the entrepreneur‘s share of the company is less than 100% after funds are raised and therefore the entrepreneur‘s incentive may be different than it would be under reward-based crowdfunding.[7] Hence, we consider a situation where the cost of production also includes the entrepreneur‘s own effort. We assume that this effort costs eq. Following similar calculations to those in the previous subsection, one can see that under reward-based crowdfunding $p_1 = p_2 = \frac{a+c+e}{2}$ and the entrepreneur‘s profit equals

$$\frac{(a-c-e)^2}{2} - I \tag{8}$$

Under equity-based crowdfunding the results may be different because of the entrepreneurial moral hazard resulting from the reduced equity stake.

The timing of events is as follows:

1. Firm selects α and sells it for price M.
2. Firm selects p_1.
3. Firm selects p_2.

[7] This is a classical moral hazard idea (Jensen and Meckling, 1976).

Proposition 1. *1) If* $\frac{(a-c-e)^2}{2} \geq I$*, the firm prefers reward-based crowdfunding; 2) Prices are higher and the quantity produced is lower under equity-based crowdfunding than under reward-based crowdfunding.*

Proof. See Appendix.

As shown in the Appendix, $p_1 = p_2 = \frac{a+c+e/(1-\alpha)}{2}$. Under equity-based crowdfunding, the price is higher than it is under reward-based crowdfunding. This is intuitive because the entrepreneur reaps less than 100% of the benefits from increasing production while bears a non-shared extra-cost, therefore, the entrepreneur chooses a lower level of production.

It is also shown that the entrepreenur's profit over the two periods equals

$$\frac{(a-c)^2}{2} - I - \frac{e^2}{2(1-\alpha)^2} + \frac{e^2}{1-\alpha} - ea + ec \tag{9}$$

If $\alpha = 0$, (9) will be equal to $\frac{(a-c-e)^2}{2} - I$. It was mentioned above that it would be the same value as it would be in the case of reward-based crowdfunding. When α is positive, the entrepreneur's profit under equity crowdfunding will be smaller since the derivative of (9) in α is negative. It is consistent with the idea of agency cost.

4 Asymmetric information about cost

So far we assumed that investors have the same information as entrepreneurs. Now suppose that the firm can be either a low-cost (high-efficiency) producer (denoted l) or a high cost (low-efficiency) producer (denoted h). More specifically, suppose that c is either equal to c_l or c_h and $c_l < c_h$. Initially the firm's type (the value of c) is determined and becomes known to the entrepreneur.

The timing of events is as follows:

1. The firm's type is revealed to the entrepreneur.
2. Firm selects financing strategy: reward-based crowdfunding or equity-based crowdfunding.
3. If equity-based crowdfunding is selected, α is determined and the firm sells it for price M. If $M < I + cq_1$, the firm is liquidated.
4. Firm selects p_1.
5. Firm selects p_2.

An equilibrium is defined as a situation where no firm type has an incentive to deviate. Since private information only concerns the production cost and not the demand side, the informational game will only affect the equity-crowdfunding scenario.[8] The price that potential investors will be paying for a fraction of

[8] We mostly focus on separating equilibria since it generates meaningful empirical implications. Further reserach is required regarding the exisitence and implications of pooling equilibria.

a firm's shares depends on their beliefs about the firm's production cost. The information game does not affect the outcome of reward-based crowdfunding. Firms will select their prices as in the case with perfect information and demand will be determined by the demand functions that are publicly known in this scenario.[9] This leads to the point that if a separating equilibrium exists, it will not be one where the high-efficiency type chooses equity-based crowdfunding since it will always be mimicked by the low-efficiency type. This result is typical for basic models with asymmetric information beginning with Akerloff (1970).

Proposition 2. *If I is sufficiently small ($\frac{(a-c_l)^2}{4} \geq I$), an efficient separating equilibrium exists, where type l selects reward-based and type h selects profit-sharing. An efficient separating equilibrium where type h selects reward-based and type l selects profit-sharing does not exist.*

Proof. See Appendix

5 Asymmetric information about demand

In this section, asymmetric information concerns the quality of a firm's products and services. In particular, we assume that, unlike outside investors, firm owners know the value of parameter a in the demand function. In the setup discussed in the previous section, a low-quality firm will always have an incentive to mimick a high-quality firm when the latter uses equity-based crowdfunding. Intutively, a similar engine should drive the results if the asymmetric information regards the product's quality rather than its cost. In order to obtain new results, we introduce new strategies. In particular, if the firm selects reward-based crowdfunding, it has two options: KIA (keep-it-all) or AON (all-or-nothing). If AON is selected, a threshold T is set, $T > 0$. If the amount of funds raised in period 1 is less than T, the firm is liquidated. We also assume that the demand is as follows: $q_t = \delta_t(a - p_t)$, where $\delta_t = 1$ with probability π and 0 with probability $1 - \pi$. Making the demand function stochastic or risky will allow us to see the role of AON method of crowdfunding ("signalling by risk-bearing").[10] Note that some empirical research suggests that many crowdfunding projects attract very low or negligeably small amounts of funds (see, for example, Mollick (2014), Cordova, Dolci and Gianfrate (2015) and Desjardins (2016)). δ_1 becomes known after the project is created and the crowdfunding method is selected. δ_2 becomes known in the beginning of period 2. Also, we assume that there are two types of firms: $a = a_h$ for type h and $a = a_l$ for type l, where $a_h > a_l$. To focus on the effect of asymmetric information, we assume, $I = 0$ (none of the results change qualitatively if $I > 0$). In particular it implies that condition (1) holds for both types of firms meaning they can use reward-based crowdfunding. Also, it means that a firm should follow the rule $p > c$ in order to accumulate

[9] In the next section we will consider a situation with asymmetric information about demand.

[10] For simplicity, previous sections did not differentiate between the different types of reward-based crowdfunding. One can easily check that it would not affect the results. The same holds with the assumption about the stochastic demand function.

sufficient funds to launch their product and avoid liquidation.

The timing of events is as follows:

1. The firm's type is revealed to the entrepreneur.
2. Firm selects financing strategy: KIA, AON or equity-based crowdfunding. If AON is selected, the firm selects T.
3. δ_1 becomes known.
4. If equity-based crowdfunding is selected, the firm selects α (fraction of shares) and sells it for an amount M.
5. Firm selects p_1.
6. If AON is selected and $p_1 q_1 < T$, the firm is liquidated.
7. If KIA or AON and $p_1 q_1 < cq_1$, the firm is liquidated. If equity-based crowdfunding is selected and $M < cq_1$, the firm is liquidated.
8. Firm's type (product's quality) becomes publicly known.
9. δ_2 becomes known.[11]
10. Firm selects p_2.

First consider the symmetric information case for KIA.

In period 2, if $\delta_2 = 1$ and $q = a - p_2$, the firm chooses p_2 to maximize $(p_2 - c)(a - p_2)$, which makes $p_2 = \frac{a+c}{2}$. If $\delta_2 = 0$ and $q = 0$, the firm's profit is zero.

In period 1, if $\delta_1 = 1$ and $q = a - p_1$, the firm maximizes $(p_1 - c)(a - p_1)$. We have $p_1 = \frac{a+c}{2}$. If $\delta_1 = 0$ and $q = 0$, the firm's profit is zero.

The firm's expected profit equals

$$\Pi = \pi \frac{(a-c)^2}{4} + \pi \frac{(a-c)^2}{4} = \frac{\pi(a-c)^2}{2} \tag{10}$$

Now consider AON. In this setting, the firm selects the pre-order price in order to maximize its sales. At the same time, it needs to reach the established threshold amount of pre-orders. In some cases it will force the firm to select a suboptimal pricing policy and in some cases (when the initial investment is sufficiently large), the project will not be successful. Also, bankruptcy is unavoi

dable under AON, if the demand is zero.

In period 2, if $\delta_2 = 1$ and $q = a - p_2$, the firm chooses p_2 to maximize $(p_2 - c)(a - p_2)$, which makes $p_2 = \frac{a+c}{2}$. If $\delta_2 = 0$ and $q = 0$, the firm's profit is zero.

[11] In this section, the timing of information revelation about a product's quality and demand is the same for any type of crowdfunding. In Section 7, we analyze the differences between different types of crowdfunding in terms of their ability to affect product's quality.

In period 1, the firm chooses T and p_1 to maximize Π where $\Pi = \pi((p_1 - c)(a - p_1) + \pi\frac{(a-c)^2}{4})$ if $p_1 q_1 = p_1(a - p_1) \geq T$.

$\Pi = 0$ if $p_1 q_1 = p_1(a - p_1) < T$.

The solution is any T such as $T \leq p_1(a - p_1)$ where $p_1 = \frac{a+c}{2}$. It does not avoid liquidation if demand is zero in period 1 but it optimizes the price policy if demand is positive.

The firm's expected profit equals

$$\Pi = \pi(\frac{(a-c)^2}{4} + \pi\frac{(a-c)^2}{4}) = \frac{\pi(1+\pi)(a-c)^2}{4} \tag{11}$$

This is smaller than (10) because under AON, bankruptcy will occur in period 1 if the amount of raised funds is smaller than T.

Finally, consider equity-based crowdfunding. In period 2, if $\delta_2 = 1$ and $q = a - p_2$, the firm chooses p_2 to maximize the entrepreneur‘s profit $(1 - \alpha)(p_2 - c)(a - p_2)$, which makes $p_2 = \frac{a+c}{2}$. If If $\delta_2 = 0$ and $q = 0$, the firm's profit is zero. The firm's expected profit in period 2 is $\frac{\pi(a-c)^2}{4}$.

In period 1, if $\delta_1 = 1$ and $q = a - p_2$, the firm chooses α and p_1 to maximize the entrepreneur's profit:

$$(1 - \alpha)(p_1(a - p_1) + M - cq_1) \tag{12}$$

subject to

$$M \geq cq_1 \tag{13}$$

The funders‘ expected earnings over the two periods should cover their investment cost or:

$$\alpha(p_1(a - p_1) + \frac{\pi(a-c)^2}{4}) \geq M \tag{14}$$

Under the optimal solution the conditions (13) and (14) will be binded because the firm can always make α as small as necessary to satisfy them. Then we have:

$$\alpha = \frac{cq_1}{p_1(a - p_1) + \frac{\pi(a-c)^2}{4}}$$

Substituting this into (12) and using the fact that if $\delta_1 = 0$ and $q = 0$, shares are not sold and the firm's profit in period 1 equals 0, we find that the entrepreneur‘s expected profit over the two periods equals:

$$\pi(p_1 - c)(a - p_1) + \frac{\pi(a-c)^2}{4}$$

This implies that $p_1 = \frac{a+c}{2}$.

The entrepreneur's expected profit then equals

$$\frac{\pi(a-c)^2}{4} + \frac{\pi(a-c)^2}{4} = \frac{\pi(a-c)^2}{2} \tag{15}$$

As we can see, this is the same amount as in (10).

Now consider asymmetric information.

Proposition 3. *If* $(\frac{a_l-c}{a_h-c})^2 < \pi < 2-(\frac{a_h-c}{a_l-c})^2$, *a separating equilibrium exists, where type l selects keep-it-all and type h selects all-or-nothing or equity-based crowdfunding. An equilibrium where type h selects keep-it-all or equity-based crowdfunding does not exist.*

Proof. See Appendix.

The right side of the inequality in Proposition 3 puts an upper bound on the probability of bankruptcy. The intuition behind this result is as follows. AON is very costly if the probability that the demand is absent is relatively high. In this case the low-quality firm will not mimick the high-quality firm. If, on the contrary, π is very large, the values of (10), (11) and (15) do not differ significantly for the low-quality firm (they are equal in the extreme case when $\pi = 1$) which means that the low-quality firm would mimick the high-quality firm and benefit from the market's optimistic belief about the quality of firms that use AON. The left side of the inequality in Proposition 3 places a lower bound on the probability of bankruptcy. If, on the contrary, the probability that demand is absent is very high, it would be beneficial for the high-quality firm to not use AON and deviate to KIA or equity-based crowdfunding.

6 Bankruptcy costs and bank monitoring

In this section we compare crowdfunding with bank financing. If the firm takes a bank loan and it is not able to pay back its debt then the firm is bankrupt and can be liquidated. On the other hand, banks have a better ability to monitor and control entrepreneurs.[12] So we assume that the manager (managerial team) has some private benefits b from each unit produced at the expense of the firm when the firm uses crowdufunding. To simplify the calculations related to bankruptcy we assume that the production output is stochastic in period 1 and depends on parameter Q (similar to stochastic demand in Section 5): $Q = 1$ with probability γ and 0 with probability $1-\gamma$. This implies that bankruptcy will only occur if the firm takes a bank loan and $Q = 0$. In contrast to firm liquidation cases when the required financing is not raised, bankruptcy does not occur as a result of failed production if the firm uses crowdfunding.[13] For simplicity assume $I = 0$. This implies that the condition $\frac{(a-c-b)^2}{4} \geq I$ holds for both firm types, which implies that crowdfunding is feasible for each type (similar to Section 3, formula (8)).

The timing of events is as follows:

[12] See, for example, Diamond (1984).

[13] In most countries there is no formal regulation that can be used to force a company into bankruptcy in the case of crowdfunding (see, for example, Gabison (2015) or Moores (2015)). There is difference, however, between equity-based and reward-based crowdfunding. If the firm uses reward-based crowdfunding then the consumers are under consumer protection law etc. (Gabison (2015)). We consider this aspect in Section 7. Here we assume that in contrast to traditional bank financing there is no bankruptcy in the case of crowdfunding. For simlicity it is assumed that the firm uses equity-based crowdfunding.

1. Firm selects financing strategy: bank loan or crowdfunding.
2. Firm selects p_1.
3. Q becomes known. If $Q = 0$ and bank loan was selected, the firm is bankrupt.
4. Firm selects p_2.

Proposition 4. *1) Prices are higher and quantity produced is lower under crowdfunding; 2) For given values of a and γ, there exists a b^* such that the firm chooses to take a bank loan if $b \geq b^*$ and chooses crowdfunding if $b < b^*$. For given values of a and b, the firm chooses to take a bank loan if γ is sufficiently large.*

Proof. See Appendix.

We find that the product price under bank financing is $p = \frac{a+c/\gamma}{2}$ and $p = \frac{a+(b+c)/\gamma}{2}$ under crowdfunding**.** Prices are higher and quantity produced is lower under crowdfunding because of the extra-cost related to moral hazard issues. The second part of the proposition states that crowdfunding will be preferred if the cost related to the absence of monitoring is relatively small. Otherwise, a bank loan will be preferred. Interestingly, the effect of a change in the probability of bankruptcy is not as straightforward as the effect of b. If the probability of bankruptcy is close to zero then a bank loan will definitely be preferred because of the monitoring advantage. However, in the middle range of the values for γ, one may find that an increase in γ benefits crowdfunding more than a bank loan. The reason for this follows from the price formulas above: a small γ amplifies the firm's moral hazard issues making the price further from optimal.

7 Hybrid cases

Ideally, the next step would be to analyze optimal financing policy when many factors such asymmetric information, moral hazard, market feedback etc. are present in the model simultaneusly. This is an intriguing challenge for future research. One should say that the creation of such a universal global model is technically difficult and in many cases may not bring many analytical and intuitively sound results.[14] This section provides an example of such an analysis.

[14] A good example is capital structure theory. Most intuitions published in textbooks for the last 50 years are based on models that consider each factor separately (pecking order theory for asymmetric inofrmation, trade-off thoery for taxes and bankruptcy costs etc.). For an example of capital structure theory review and the role of market imperfections see Harris and Raviv (1991), Miglo (2011) and Miglo (2016). Models combining several factors are much less popular and much more technically complicated though some researchers suggest that these types of models are a prominnet direction for future research. Also note that based on managers' surveys, managers only support around 50% (see, for example, Graham and Harvey (2001)) of basic theories, which means that the precentage of managers that use even more complicated ideas is even smaller. Crowdfunding theory is a much younger theory than

Case 1. Consider the situation where firms have private information about production costs (Section 4). In this situation reward-based crowdfunding can be used as a signal of a firm's quality. Now suppose that a firm is terminated (bankrputcy occurs) in period 1, if the firm is not able to deliver its product to customers and the firm uses reward-based crowdfunding (similar to the ideas from sections 5 and 6). Gabison (2015) noted[15] that eventhough there is no formal regulation of reward-based crowdfunding in most countries, in most cases consumers (funders) are under consumer protection law (which exists in most developed countries) and therefore a violation of this law can be costly for the firm. As in Section 4, c is either equal to c_l or c_h and $c_l < c_h$. Like in Section 6, the production output is stochastic in period 1 and depends on parameter Q (similar to stochastic demand in Section 6): $Q = 1$ with probability γ or 0 with probability $1 - \gamma$. Bankruptcy only occurs when $Q = 0$ and the firm uses reward-based crowdfunding. Bankruptcy does not occur as a result of failed production in period 1 under equity-based crowdfunding since by its nature no promises are made to funders/investors and dividends are not guaranteed. Like in Section 5, for simplicity we assume $I = 0$.

The timing of events is as follows:

1. The firm's type is revealed to the entrepreneur.
2. Firm selects financing strategy: reward-based crowdfunding or equity-based crowdfunding.
3. If equity-based crowdfunding is selected, α is determined and the firm sells it for price M.
4. Firm selects p_1.
5. If equity-based crowdfunding is selected and $M < cq_1$, the firm is liquidated. If reward-based crowdfunding is selected and $p_1 q_1 < cq_1$, the firm is liquidated.
6. Q becomes known. If $Q = 0$ and reward-based crowdfunding was selected, the firm goes bankrupt.
7. Firm selects p_2.

An equilibrium is defined as a situation where no firm type has the incentive to deviate. Like in Section 4, since information only concerns the production cost and not the demand side, the informational game will only affect the equity-crowdfunding scenario.

capital structure theory so it is in the stage of its development where the quality and relative simplicity of its basic ideas are probably the most important objectives of its research along with managerial education on these ideas (see, for example, Loane, Ramsey and Ibbotson (2016)).

[15] See also Ibrahim (2016) and Moores (2015) for a legal environment analysis regarding reward-based crowdfunding. Mollick (2015) empirically analyzes the percentage of failed firms that used reward-based crowdfunding.

Proposition 5. *If* $\frac{a-c_l}{a-c_h} < 2$, *a separating equilibrium does not exist. Otherwise, if* γ *is sufficiently large, the only efficient separating equilibrium that exists is one where type* l *selects reward-based crowdfunding and type* h *selects profit-sharing.*

Proof. See Appendix.

To explain the results of this proposition, note that Section 4 found that high-quality firms can use reward-based crowdfunding to signal their quality. That section did not consider a potential cost of reward-based crowdfunding related to bankruptcy in the case when the firm is not able to deliver their product in period 1. This case asks if the result stands if such a cost is taken into consideration. What we found is that the result stands but there are cases when a separating equilibrium where a high-quality firm uses reward-based crowdfuning does not exist. The meaning of the condition stated in the proposition is that if the difference between the firm types is sufficiently small, such an equilibrium may not exist. Secondly and more interestingly is that if the probability of bankruptcy is sufficiently small, an equilibrium may not exist. In this case, a low-quality firm may still be interested in mimicking a high-quality type when the latter choses reward-based crowdfunding.

Case 2. Similar to some previous sections, this case considers a model with imperfect information. However, here we assume that crowdfunding helps the firm obtain information about demand. Suppose that if the firm uses crowdfunding, it can improve the product's quality after obtaining useful information about demand in period 1: more specifically, in period 2 the demand becomes $q = sa - p, s \geq 1$.[16] We assume that s has different values for different types of crowdfunding: $s \in \{s_r, s_e\}, s_r > s_e$ where s_r is the product improvement if reward-based crowdfunding is used. $s_r > s_e$ because under reward-based crowdfunding, the funders know that the firm's launch of production and, respectively, its survival depend on their pre-orders and the firm's response to this feedback is expected to be very efficient since the firm's survival depends on it.[17] Also, under reward-based crowdfunding, the funders have a short-term interaction with the firm whereas under equity-based crowdfunding, these interactions are long-term. So the former incentivizes the funders to provide a more intense feedback. If the firm uses traditional financing like a bank loan, for example, it does not get the same feedback as it would with crowdfunding and the demand does not change in period 2. On the other hand, as in Section 6, banks have a

[16] Xu, Yang, Rao, Fu, Huang, and Bailey (2014), Block, Hornuf and Moritz (2016) and da Cruz (2016) empiricaly analyze different aspects of the informational value of crowdfunding for entrepreneurs.

[17] Note that the market feedback represents probably the most important community benefit of crowdfunding for the firm (because it may increase its product quality and repsectively their future profits) as well as for funders and customers who can enjoy higher quality products as a result of market feedback. Note also that we explicity model this mechanism in our model through providing better information to the firm in period 1, which allows them to improve their product's quality in period 2 etc. Belleammey et al (2014) assume that there are some exogenously given community benefits in period 1 as a result of crowdfunding. Note that Cholakova and Clarysse (2015) find that non-monetary benefits do not play a singificant role for funders.

better ability to monitor and control the entrepreneurs.[18] We assume that the manager has some private benefits b when using crowdfunding.

The timing of events is as follows:

1. Firm selects a financing strategy: bank loan, reward-based crowdfunding or equity-based crowdfunding.
2. If equity-based crowdfunding is selected, the firm chooses α (the fraction of the firm for sale) and sells it for price M. If $M < I + cq_1$, the firm is liquidated.
3. Firm selects p_1. The demand for the product is determined.
4. Firm selects p_2. The demand for the product is determined.

Proposition 6. *For a given value of a, if I is sufficiently small, the firm takes a bank loan if s_r is sufficiently small or b is sufficiently large. Otherwise, the firm selects reward-based crowdfunding. If I is sufficiently large, the firm takes a bank loan if s_e is sufficiently small or b is sufficiently large. Otherwise, the firm selects equity-based crowdfunding. Prices are higher and quantity produced is lower under crowdfunding. For a given value of I, crowdfunding is selected over a traditional bank loan if a is either very small or very large. For medium levels of a, a bank loan is preferred.*

Proof. See Appendix.

It shown in the Appendix that the entrepreneur's profits under the different strategies are equal to the following.

$$\Pi_r = \frac{(a-b-c)^2}{4} + \frac{(s_r a - b - c)^2}{4} - I$$

$$\Pi_e = \frac{(a-b-c)^2}{4} + \frac{(s_e a - b - c)^2}{4} - I$$

$$\Pi_b = \frac{(a-c)^2}{2} - I$$

where subscript r stands for reward-based crowdfunding, e means equity-based crowdfunding and b means bank loan.

The firm is indifferent between reward-based crowdfunding and a bank loan if

$$\frac{(a-b-c)^2}{2} + \frac{(s_r a - b - c)^2}{2} = (a-c)^2 \tag{16}$$

The firm is indifferent between equity-based crowdfunding and a bank loan if

$$\frac{(a-b-c)^2}{2} + \frac{(s_e a - b - c)^2}{2} = (a-c)^2 \tag{17}$$

[18] Other traditional forms of entrepreneurial financing such as venture capital financing also have a high degree of monitoring so the model can be applied to those cases as well.

Also if

$$(a - c - b)^2 < 4I \tag{18}$$

the firm will not be able to use reward-based crowdfunding. And if $(a-c-b)^2 \geq 4I$, the firm prefers reward-based crowdfunding over equity-based crowdfunding.

Figure 1 illustrates the equilibrium decision-making for the entrepreneurs. The lines represent equations (16), (17) and (18). Letters RC, EC and B denote the areas where the entrepreneurs choose reward-based crowdfunding, equity-based crowdfunding, and a bank loan respectively.

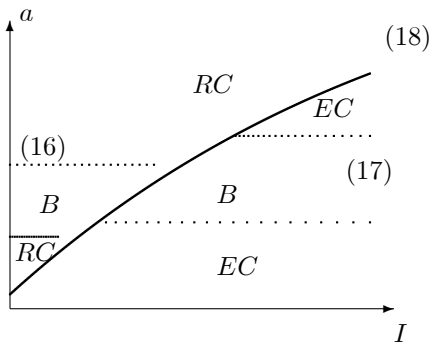

Figure 1. The choice of financing.

As follows from Figure 1, firms that use crowdfunding are either projects with very small demand or very high demand. Also, entrepreneurs with EC have higher I for any value of a compared to entrepreneurs with RC. Overall we can see that firms with a medium level of demand prefer B, firms with stronger demand prefer crowdfunding, firms with a large amount of investments and strong demand or very weak demand prefer EC and firms with smaller investments and strong demand or very weak demand prefer RC.

8 Implications

Our paper has several implications for an entrepreneurial firm's choice of financing. The summary of the results is presented in Table 1.

Market imperfection(s)	*Results*
Asymmetric information about production cost	Good quality projects prefer reward-based crowdfunding
Entrepreneur's moral hazard due to his reduced equity stake	Firms prefer reward-based crowdfunding. Prices are higher and quatity produced is lower under equity-based crowdfunding
Asymmetric information about demand	*"Signalling by risk-bearing".* Low-quality firm selects KIA or equity-based crowdfunding and high-quality firm selects AON
Bankruptcy costs vs. bank monitoring	Prices are higher and quantity produced is lower under crowdfunding than under a bank loan
Hybrid case 1 (asymmetric information about demand and bankruptcy costs)	If bankrupcy costs are high, firms use reward-based crowdfunding to signal their quality
Hybrid case 2 (market feedback, bank monitoring)	Prices are higher and quantity produced is lower under crowdfunding. Crowdfunding is selected over a traditional bank loan if demand is either very small or very large.

Table 1. Market imperfections and the model's results.

Proposition 2, 3 and 5 imply that when asymmetric information is important, high-quality projects prefer reward-based crowdfunding. This is contradictory, to some extent, to the spirit of the results in Belleammey et al (2014), which finds that asymmetric information favors equity-based crowdfunding.[19] Note, however, that the objective of their analysis is different from ours. For example, they do not analyze the case when the decision about the choice of crowdfunding mode is part of the model (this is obviously a crucial part of our model; consequently they automatically do not consider the possibility that firms can signal their quality with their choice of crowdfunding) so they only compare the symmetric and asymmetric information cases within each type of crowdfunding. Also, it is mentioned in Belleammey et al (2014) that their analysis of the asymmetric information case is not complete.[20] In our model, equity-based crowdfunding suffers more from asymmetric information, which is consistent with the spirit of the majority of finance literature where equity-financing is

[19] For example, it is well-known in capital structure theory that asymmetric information damages equity financing more than debt financing and that equity financing can not be used by a high-quality type as a signal of quality whereas in some cases debt financing can be used (Leland and Pyle (1977)). So applying this example to Belleamey et al (2014), who claim that asymmetric information is more damaging for reward-based crowdfunding, it would be no surprise to find that a separating equilibrium where a high-quality firm uses reward-based crowdfunding does not exist or that there is a separating equilibrium where the high-quality firm uses equity-based crowdfunding.

[20] Among other things note, for example, that the proof of Lemma 5, which is crucial for Proposition 2, relies on numerical simulations, Section 4.2.2 is not finished and, as mentioned above, the case when the decision about the choice of crodwfunding mode is part of the model is not analyzed.

generally the most sensitive to the asymmetric information problem. Equity-based crowdfuning cannot be used as a signalling tool by a high-quality firm since it will always be mimicked by a low quality firm as the share price of a high-quality firm is always higher than that of a low-quality firm. In contrast, a high-qulity can use reward-based crowdfunding. This is because a low-quality firm may find it unproftable to mimick this strategy as it will be taking more risk to achieve its threshold. This prediction has not been directly tested but is consistent with the spirit of the results found in Ahlers, Cumming, Guenther, and Schweizer (2015) and Mollick (2014) (that the firm's financing choice can serve as a signal of a project's quality). Furthermore, the entrepreneur's larger fraction of equity is associated with a higher project quality. In our case, reward-based crowdfunding implies a higher fraction of ownership held by the entrepreneur. Ahlers et al (2015) examine the effectiveness of the signals used by entrepreneurs to induce (small) investors to commit financial resources in an equity-based crowdfunding context. They found that retaining equity is an effective signal and can therefore strongly impact the probability of a funding's success. It is consistent with the spirit of our result that reward-based crowdfunding may be preferred by entrepreneurs of higher quality.

Proposition 3 implies that high-quality projects may prefer AON over KIA. This is consistent with the spirit of Cumming, Leboeuf and Schwienbacher (2014). They show that KIA campaigns are less successful in meeting their fundraising goals. Also, note that the rate of success of campaigns on Kickstarter, which only uses AON, is higher than on Indiegogo.[21]

Proposition 1 and 4 imply that pricing and production strategies are affected by moral hazard issues and the costs of financial distress. In particular, prices can be higher and quantity produced can be lower under equity-based crowdfunding. This is consistent with Paakkarinen (2016) that noted that in contrast to pre-ordering, profit sharing may have fewer customers, but higher margins. More broadly, the point that moral hazard issues related to the entrepreneurial cost of effort and the reduced equity stake are more important, under equity-based crowdfunding is consistent with Gabison (2015) and Paakkarinen (2016), which noted that equity-based crowdfunding is much more constricted in comparison to other forms of crowdfunding.

As follows from Moores (2015), the bankruptcy procedure is not clearly defined in the case of a failed crowdfunding campaign, in fact, the firm may not even be declared bankrupt even though consumers are under the customer protection law (at least in the case of reward-based crowdfunding). As noted in Moores (2015), further development and clarifications in this area are helpful. Our analysis suggests that from a policy perspective higher bankruptcy costs are benefitial for the exsitence of separating equilibria where high-quality firms can use reward-based crowdfunding to singal their quality and avoid being mimicked by low-quality firms.

As follows from Proposition 6, firms should avoid crowdfunding if moral

[21] See, for example:
http://crowdfunding.cmf-fmc.ca/facts_and_stats/how-likely-is-your-crowdfunding-campaign-to-succeed

hazard considerations related to the weak ability of funders to monitor the firm (compared to traditional financing from bank loans or venture capital financing) are very important. These results are consistent with Xu (2017) that finds that entrepreneurs swich between crowdfunding an bank borrowing depending on the relative costs of financing. Also, and perhaps more interestingly is that if we only consider reward-based crowdfunding vs. bank financing (the area above line (18) in Figure 1), projects with high I and high a, i.e potentially high risk, high investment (novelty) and potentially high demand (a) will prefer crowdfunding vs. bank financing. This is also consistent with Xu (2017). Finally, we find that firms should use crowdfunding for either projects with a very small demand or a very high demand. Also, firms that use equity-based crowdfunding have a higher amount of fixed costs compared to entrepreneurs with reward-based crowdfunding.

In Belleammey et al (2014) price discrimination is not possible in the absence of non-monetary benefits, and therefore both forms of crowdfunding yield exactly the same outcome as seeking money from a bank or a large equity investors. Some research discovered however that the role of such non-monetary benefits in crowdfunding is negligeable (see, for example, Cholakova and Clarysse (2015)). In our model, there are no non-monetary benefits from crowdfunding but the benefits of crowdfunding (compared to traditional financing) arise from natural features of crowdfunding such as market feedback. Note that overall, the focus of most existing theoretical papers on cowdfunding has been to exploit features of crowdfunding like the opportunity for the entrepreneur to price discriminate. However, recent literature finds empirically that crowdfunding also has a lot of informational value for entrepreneurs. Hence, our article mostly focuses on the latter aspect of crowdfunding.

Finally, note that from Lemma 1, large projects, in most cases, prefer equity-based crowdfunding. As mentioned previously, in our case this is not due to the presence of financial market imperfections but to the fact that funders can count on long-term firm profits in the case of equity-based crowdfunding. As mentioned previously, this result in Belleammey et al (2014) is due to the assumptions about community benefits in period 1 when firm conducts crowdfunding. These benefits differ among funders in the case of a reward-based campaign so the small size of the crowdfunding allows the firm to capture these differences very efficiently, while in the equity-based case community benefits are more uniform so there is no advantage of having a small scale. As follows from Paakkarinen (2016), equity-based campaigns are much larger than reward-based campaigns but firms select equity-based campaigns mostly for possibility of collecting a large amounts of capital and not to select a better price discrimination approach.

9 The model extensions and robustness

Different demand functions. Our focus in this article is to analyze the role of different market imperfections in crowdfunding. That is why we adopt a relatively simple demand function. In dynamic monopoly pricing literature

this approach is not unusual (see, for example, Demichelis and Tarola (2006)). Most of our results (such as Propositions 1, 2 etc.) are intuitively sound and will hold if mathematically different demand functions are used. Alternatively, a sigificantly different approach of modelling the demand side can be taken where individual customers with different demand functions are included (see, for example, Belleammey et al (2014) and Hu, Li and Shi (2014)). This approach is often used in industrial organization or price discrimination literature. Our focus is on market imperfections and financial aspects of crowdfunding and the approach that uses total demand functions from investors/funders (the market) is very common.[22] Note also that Belleammey et al (2014) make the ad-hoc assumption that crowdfunding provides an automatic benefit to funders.

Different types of moral hazard. In our model (Section 3), the entrepreneurial moral hazard takes place because the entrepreneur's equity stake in the firm is reduced while his individual effort is costly and this cost is not shared. This approach is very common in financing literature (starting with Jensen and Meckling (1976)) and typically creates an agency cost of equity financing as in our paper. There are many different ways to analyze moral hazard issues, for example, to explicitely model the entrepreneur's level of effort. This apaproach is quite common in contract literature. In finance literature this approach was used, for example, in Innes (1991). The result of that analysis reveals the advantage of debt financing over equity financing which is consistent with the spirit of our modelling where equity-based crowdfunding has a disadvantage due to entrepreneurial moral hazard. In Section 6 we again use moral hazard to compare crowdfunding and bank loans using the idea that bank financing provides better monitoring. This idea is standard in finance literature (see, for example, Diamond (1984)).

The distribution of types. In sections 4 and 5, which deal with asymmetric information we use two types of firms to illustrate the main ideas. This is also very typical in literature. A natural question though is whether the results stand if one considers a case with multiple types. Our analysis shows[23] that most conclusions remain the same: under asymmetric information, equity-based crowdfunding is an inferior choice compared to reward-based crowdfunding. In the case of multiple types, however, one may have a semi-separating or even pooling equilibrium where only the type with the highest cost (speaking about Section 4) will be indifferent between the two types of crowdfunding and all other types select reward-based crowdfunding. In Section 5, our analysis shows

[22] One can further discuss the similarity between the two approaches. One can see, for example, that in the spirit of that literature our model can be interpretted as a case with one customer in each period without the possibility of product substitution between periods. One can see though that if substitution is allowed between periods, most results would stand since the period 2 product price is not less than the period 1 price in most cases so it makes no sense for this customer to wait until period 2 to purchase the product. Introducing numerous customers with different product valuations will definitely complicate the model, however, most intutions in this paper will not be affected.

[23] Proofs are available upon demand. Note that the calculations become much longer and technically more complicated, which is very typical for multiple types games with asymmetric information.

that the results may hold even in a multiple types environment though more research is required. The main implication of our analysis holds. In particular, our results show that there is no semi-separating equlibrium where the average quality of types that choose equity-based crowdfunding or the KIA method is higher than those that choose AON, which is consistent with our basic model.

Mixed financing and more types of financing. Unlike capital structure literature, where debt/equity mix is a very common strategy (as opposite to pure equity or pure debt financing), simultaneously conducting different kinds of crowdfunding is not common. Nevertheless, if mixed financing is allowed in period 1, most results will stand. For example, if mixing bank debt and crowdfunding is allowed in period 1, as in Section 2, the results stand though the condition (1) can be softened for a firm if it uses equity-based crowdfunding. Similarly, Proposition 1 stands qualitatively but the formulas will be quantitaively different. In Sections 3 and 4, a signalling equilibrium may still exist where a high-quality firm uses a mix of reward-based crowdfunding and a bank loan or a mix of a bank loan and AON, as in Section 4, although restricting conditions will change quantitatively. Introducing additional financing strategies such as debt-based crowdfunding is an interesting direction. Most resutls regarding the costs and benefits of different financing strategies found in this paper are quite general and do not depend on introduction of more options in the model. Quantitatively though, some conditions may change. It is definitely an interesting direction for future research. Note that most existing theoretical literature on crowdfunding does often consider reward-based and equity-based crowdfunding separately from debt-based crowdfunding. One of the reasons for this seems to be that the founders' objectives are quite different in these scenarios (see, for example, Hildebrand, Puri, and Rocholl (2014)).

10 Conclusions

Most existing theoretical papers on crowdfunding consider static models.[24] This paper is one of the first papers that analyzes a dynamic (two-period) model of crowdfunding. Existing theoretical literature on crowdfunding has extensively focused on such features of crowdfunding as price discrimination. This paper is one of the first that focuses on information aspects of crowdfunding, which is more in the spirit of finance literature than industrial organization literature. In particular, this is one of the first papers that obtains analytical results for models with asymmetric information. Most existing literature focuses more on moral hazard issues. Also, this paper is one of the first that analyzes the choice between different types of crowdfunding (reward-based vs. equity-based) and the choice between crowdfunding and traditional financing. In addition to traditional forms

[24] Technically there are two periods in Belleammey et al (2014) but only one period of production. Periods 1 and 2 in their model differ in that there is pre-ordering in period 1 (or stock sales) and production takes place in period 2. In our model, pre-ordering and stage 1 production happen in the same period. The presence of two production periods allows us to capture an essential difference between reward-based and equity-based crowdfunding: under equity-based crowdfunding funders can count on long-term firm profits.

of markets imperfections (asymmetric information, moral hazard, bankruptcy costs etc.) our model includes some other features of crowdfunding such as market feedback. The model provides several implications, most of which have not been yet been tested. When asymmetric information is important, high-quality projects prefer reward-based crowdfunding. The choice of the all-or-nothing mechanism as opposed to keep-it-all can serve as a signal of a firm's quality. Finally, crowdfunding is selected over a traditional bank loan if the demand for the product is either very small or very large.

Appendix

Proof of proposition 1.

In period 2, the firm chooses p_2 to maximize the entrepreneur's profit

$$(1-\alpha)(p_2-c)(a-p_2)-e(a-p_2) \tag{19}$$

, which makes $p_2=\frac{a+c+e/(1-\alpha)}{2}$.

In period 1, after shares are sold, the firm chooses p_1 ($q_1=a-p_1$) to maximize

$$(1-\alpha)(p_1(a-p_1)+M-I-cq_1)-e(a-p_1) \tag{20}$$

subject to $M\geq I+cq_1$. Two cases are possible. If $\frac{a+c+e/(1-\alpha)}{2}\geq\frac{I-M}{c}+a$ we have $p_1=\frac{a+c+e/(1-\alpha)}{2}$. Otherwise we have a corner solution $p_1=\frac{I+ca-M}{c}$. In both cases, under the optimal strategy chosen by the firm $M=I+cq_1$.

The funders anticipate it and therefore M and α will be connected as follows:

$$M=\alpha(p_1(a-p_1)+(p_2-c)(a-p_2))=I+c(a-p_1) \tag{21}$$

Then we have:

$$\alpha=\frac{I+c(a-p_1)}{p_1(a-p_1)+(p_2-c)(a-p_2)}$$

Substituting this into (19) and (20) we get that the entrepreneur's expected profit over the two periods equal to:

$$(p_1-c-e)(a-p_1)-I+(p_2-c-e)(a-p_2) \tag{22}$$

In the beginning of period 1, the entrepreneur selects α to maximize (22). The case where $\frac{a+c+e/(1-\alpha)}{2}<\frac{I-M}{c}+a$ is not optimal. The firm should increase M and α becasue of the following. (22) is concave in p_1 and $p_1=\frac{a+c+e}{2}$ is an optimal p_1 in (22). Further $p_1=\frac{a+c+e/(1-\alpha)}{2}$ is closer to the optimum than $p_1=\frac{I+ca-M}{c}$. So we have $p_1=p_2=\frac{a+c+e/(1-\alpha)}{2}$.

Using the above formulas for p_1 and p_2, (22) can be converted into

$$\frac{(a-c)^2}{2}-I-\frac{e^2}{2(1-\alpha)^2}+\frac{e^2}{1-\alpha}-ea+ec \tag{23}$$

If $\alpha = 0$, (23) will be equal to $\frac{(a-c-e)^2}{2} - I$. This is the same value as under a reward-based crowdfunding scenario. When α is positive, the entrepreneur's profit under equity crowdfunding will be smaller since the derivative of (23) in α is negative.

Proof of Proposition 2. Consider a situation where l selects reward-based crowdfunding and h selects profit-sharing. If I is sufficiently small, we have (based on Section 2 calculations)

$$\Pi_h = \frac{(a-c_h)^2}{2} - I \tag{24}$$

$$\Pi_l = \frac{(a-c_l)^2}{2} - I \tag{25}$$

where Π_j is the equilibrium profit of type j (all calculations are based on the symmetric information case for each type described in the previous section). Also we have (as follows from (21))

$$\alpha_h = \frac{2I + c_h(a-c_h)}{a_h(a-c_h)} \tag{26}$$

h does not have an incentive to mimick l since, as mentioned above, in this section asymmetric information does not concern reward-based crowdfunding. So if h chose reward-based crowdfunding it would have the same payoff as it would in equilibrium: $\frac{(a-c_h)^2}{2} - I$. Now suppose that l mimics h and chooses equity-based crowdfunding instead. l's profit Π_{lh} then equals

$$\Pi_{lh} = (1-\alpha_h)(p_1 q_1 + (p_{2l} - c_l)(a - p_{2l}))$$

In this equation $p_{2l} = \frac{a+c_l}{2}$ (as follows from Section 2) and α_h is determined by (26). Note that when l mimicks h, it has to sell a larger stake of equity in the firm compared to the symmetric information case. Indeed if l sells equity under symmetric information we have

$$\alpha_l = \frac{2I + c_l(a-c_l)}{a_l(a-c_l)} \tag{27}$$

This is smaller than (26) because $c_l < c_h$. Note that the amount of funds raised will be different under symmetric information. Keeping unused cash is useless so prices and quantities will be different from the symmetric information case for type l. More specifically, we have

$$q_1 = \frac{c_h(a-p_{1h})}{c_l}$$

$$p_1 = a - q_1$$

Indeed, $I + c_h q_h = I + c_h(a - p_{1h})$ is the amount of funds raised for selling shares. From this amount, I will cover the fixed costs for l. The remaining

amount $c_h(a-p_{1h})$ will be used to cover the variable costs of production, which are equal to c_l per unit for type l. Also

$$p_{1h} = \frac{a + c_h}{2}$$

It implies

$$\Pi_{lh} = (1 - \frac{2I + c_h(a - c_h)}{a_h(a - c_h)})((a - \frac{c_h(a - c_h)}{2c_l})\frac{c_h(a - c_h)}{2c_l} + \frac{(a - c_l)^2}{4})$$

This is less than (25) because $c_l < c_h$. Therefore l will not mimick h.

Now consider a situation where h selects reward-based crowdfunding and l selects profit-sharing. As before we have

$$\Pi_h = \frac{(a - c_h)^2}{2} - I \tag{28}$$

$$\Pi_l = \frac{(a - c_l)^2}{2} - I$$

Suppose that h mimics l and chooses equity-based crowdfunding instead. Using similar reasoning one can show that h's profit Π_{hl} equals

$$\Pi_{lh} = (1 - \frac{2I + c_l(a - c_l)}{a_l(a - c_l)})((a - \frac{c_l(a - c_l)}{2c_h})\frac{c_l(a - c_l)}{2c_h} + \frac{(a - c_h)^2}{4})$$

This is greater than (28) because $c_l < c_h$. Therefore h will mimick l. This means that such an equilibrium does not exist.

Proof of Proposition 3. Consider a situation where type l selects keep-it-all and type h selects all-or-nothing. First we have

$$\Pi_{1h} = \frac{\pi(1 + \pi)(a_h - c)^2}{4} \tag{29}$$

$$\Pi_{1l} = \frac{\pi(a_l - c)^2}{2} \tag{30}$$

where Π_{1j} is the equilibrium profit of type j (all calculations are based on the symmetric information case for each type described in the previous section). Suppose that l mimics h and chooses AON. We have

$$\Pi_{lh} = \pi((p_{1h} - c)(a_h - p_{1h}) + \pi\frac{(a_l - c)^2}{4})$$

where $p_{1l} = \frac{a_l + c}{2}$ and $p_{1h} = \frac{a_h + c}{2}$.

We have

$$\Pi_{lh} = \frac{\pi(a_h - c)^2}{4} + \frac{\pi^2(a_l - c)^2}{4}$$

Comparing this with (30) we find that the former is greater if

$$\pi < 2 - (\frac{a_h - c}{a_l - c})^2 \tag{31}$$

and therefore type l has no incentive to deviate.

Suppose that h mimics l and chooses KIA. We have

$$\Pi_{hl} = \pi\frac{(a_l - c)^2}{4} + \pi\frac{(a_h - c)^2}{4}$$

Comparing with (29) we find that h does not deviate if

$$\pi > (\frac{a_l - c}{a_h - c})^2 \tag{32}$$

Note that conditions (31) and (32) do not contradict each other. It is because the right side of (32) is smaller than that of (31). Indeed let $x = (\frac{a_l - c}{a_h - c})^2$. Then the following makes the comparison described in the previous sentence:

$$x < 2 - \frac{1}{x}$$

, which always holds.

Consider a situation where type h selects keep-it-all and type l selects all-or-nothing. First we have

$$\Pi_{1h} = \frac{\pi(a_h - c)^2}{2} \tag{33}$$

$$\Pi_{1l} = \frac{\pi(1 + \pi)(a_l - c)^2}{4} \tag{34}$$

where Π_{1j} is the equilibrium profit of type j (all calculations are based on symmetric information case for each type described in previous section). Suppose that l mimics h and chooses KIA. We have

$$\Pi_{lh} = \pi\frac{(a_h - c)^2}{4} + \pi\frac{(a_l - c)^2}{4}$$

This is greater than (34) because $a_h > a_l$ and $\pi < 1$. So a situation where type h selects keep-it-all and type l selects all-or-nothing is not an equilibrium.

Finally, consider a situation where type h selects equity-based crowdfunding. We have

$$\Pi_{1h} = \frac{\pi(a_h - c)^2}{2} \tag{35}$$

$$\Pi_{1l} \leq \frac{\pi(a_l - c)^2}{2} \tag{36}$$

(if l selects KIA, (36) holds as an equality). Suppose that l mimics h and chooses equity-based crowdfunding. l's profit Π_{lh} then equals

$$\Pi_{lh} = (1 - \alpha_h)(\pi p_{1l}(a_l - p_{1l}) + \pi(p_{2l} - c)(a_l - p_{2l}))$$

where:

$$\alpha_h = \frac{c}{a_h}$$

$$p_{1l} = p_{2l} = \frac{a_l + c}{2}$$

It implies

$$\Pi_{lh} = (1 - \frac{c}{a_h})\frac{\pi a_l(a_l - c)}{2}$$

This is greater than (36) because $a_l < a_h$. Therefore l will mimick h and such an equilibrium does not exist.

Proof of Proposition 4.

Consider crowdfunding. Calculations are similar to Section 4. In period 2, the firm chooses p_2 to maximize the entrepreneur's profit $(1-\alpha)(\gamma p_2 - c - b)(a - p_2)$, which makes $p_2 = \frac{a+(b+c)/\gamma}{2}$.

In period 1, after the shares are sold, the firm chooses p_1 to maximize $(1 - \alpha)(\gamma p_1(a - p_1) + M - (c + b)q_1)$ subject to

$$M \geq (c + b)q_1 \tag{37}$$

. It implies:

$$p_1 = \frac{a + (c + b)/\gamma}{2} \tag{38}$$

. The firm's expected profit in period 1 is $\gamma p_1(a - p_1) = \gamma\frac{(a+(c+b)/\gamma)(a-(c+b)/\gamma)}{4}$. The funders' expected earnings should cover their investment cost or:

$$\alpha(\gamma\frac{(a + (c + b)/\gamma)(a - (c + b)/\gamma)}{4} + \frac{\gamma(a - (c + b)/\gamma)^2}{4}) \geq M \tag{39}$$

Under optimal solution the conditions (37) and (39) will be bounded because the firm can always make α as small as necessary to satisfy them. Then we have:

$$\alpha = \frac{(c + b)q_1}{\gamma\frac{(a+(c+b)/\gamma)(a-c_h/\gamma)}{4} + \frac{\gamma(a-(c+b)/\gamma)^2}{4}} = \frac{c\frac{a-(c+b)/\gamma}{2}}{\frac{\gamma a(a-(c+b)/\gamma)}{2}} = \frac{c + b}{\gamma a}$$

The entrepreneur's expected profit over the two periods equals:

$$(1 - \frac{(c + b)}{\gamma a})(\frac{\gamma a(a - (c + b)/\gamma)}{2}) = \frac{(\gamma a - c - b)^2}{2\gamma} \tag{40}$$

Consider bank loan financing. In period 2, the firm chooses p_2 to maximize $(\gamma p_2 - c)(a - p_2)$ which makes $p_2 = \frac{a+c/\gamma}{2}$. Note that this is smaller than (38). The firm's expected profit in period 2 is $\frac{\gamma(a-c/\gamma)^2}{4}$.

In period 1, the firm maximizes $\gamma(p_1(a - p_1) + \frac{\gamma(a-c/\gamma)^2}{4}) - c(a - p_1)$ subject to: $p_1 q_1 = p_1(a - p_1) \geq cq_1 = c(a - p_1)$.

The solution gives us $p_1 = \frac{a+c/\gamma}{2}$.

The firm's profit over the two periods equals

$$\Pi = \frac{\gamma(a - c/\gamma)^2}{4} + \frac{\gamma^2(a - c/\gamma)^2}{4} = \frac{(1 + \gamma)(\gamma a - c)^2}{4\gamma} \tag{41}$$

The comparison of (40) and (41) leads to the first part of Proposition 4. In particular, (40) is decreasing, which implies that the firm prefers crowdfunding if b is sufficiently small. Also, when $\gamma = 1$, (41) is strictly greater than (40).

Proof of Proposition 5.

Consider a situation where l selects reward-based crowdfunding and h selects profit-sharing.

Consider firm h. Calculations are similar to Section 4. In period 2, the firm chooses p_2 to maximize the entrepreneur's profit $(1-\alpha)(\gamma p_2 - c_h)(a-p_2)$, which makes $p_2 = \frac{a+c_h/\gamma}{2}$.

In period 1, after the shares are sold, the firm chooses p_1 to maximize $(1-\alpha)(\gamma p_1(a-p_1) + M - c_h q_1)$ subject to

$$M \geq cq_1 \tag{42}$$

. It implies: $p_1 = \frac{a+c_h/\gamma}{2}$. The firm's expected profit in period 1 is $\gamma p_1(a-p_1) = \gamma\frac{(a+c_h/\gamma)(a-c_h/\gamma)}{4}$. The funders' expected earnings should cover their investment cost or:

$$\alpha(\gamma\frac{(a+c_h/\gamma)(a-c_h/\gamma)}{4} + \frac{\gamma(a-c_h/\gamma)^2}{4}) \geq M \tag{43}$$

Under the optimal solution the conditions (42) and (43) will be bounded because the firm can always make α as small as necessary to satisfy them. Then we have:

$$\alpha = \frac{c_h q_1}{\gamma\frac{(a+c_h/\gamma)(a-c_h/\gamma)}{4} + \frac{\gamma(a-c_h/\gamma)^2}{4}} = \frac{c_h\frac{a-c_h/\gamma}{2}}{\frac{\gamma a(a-c_h/\gamma)}{2}} = \frac{c_h}{\gamma a}$$

The entrepreneur's expected profit over the two periods equals:

$$(1-\frac{c_h}{\gamma a})(\frac{\gamma a(a-c_h/\gamma)}{2}) = \frac{(\gamma a - c_h)^2}{2\gamma} \tag{44}$$

Consider firm l. In period 2, the firm chooses p_2 to maximize $(\gamma p_2 - c_l)(a-p_2)$ which makes $p_2 = \frac{a+c_l/\gamma}{2}$. The firm's expected profit in period 2 is $\frac{\gamma(a-c_l/\gamma)^2}{4}$

In period 1, the firm maximizes $\gamma(p_1(a-p_1) + \frac{\gamma(a-c_l/\gamma)^2}{4}) - c_l(a-p_1)$ subject to: $p_1 q_1 = p_1(a-p_1) \geq c_l q_1 = c_l(a-p_1)$. The solution gives us $p_1 = \frac{a+c_l/\gamma}{2}$.

The firm's profit over the two periods equals

$$\Pi_l = \frac{\gamma(a-c_l/\gamma)^2}{4} + \frac{\gamma^2(a-c_l/\gamma)^2}{4} = \frac{\gamma(1+\gamma)(a-c_l/\gamma)^2}{4} \tag{45}$$

Suppose that l mimics h and chooses equity-based crowdfunding instead. l's profit Π_{lh} then equals

$$\Pi_{lh} = (1-\alpha_h)(\gamma p_{1l}(a-p_{1l}) - c_l(a-p_{1l}) + \gamma p_{2l}(a-p_{2l}) - c_l(a-p_{2l}))$$

where:

$$\alpha_h = \frac{c_h}{\gamma a} \tag{46}$$

$$p_{1l} = p_{2l} = \frac{a + c_l/\gamma}{2} \tag{47}$$

It implies

$$\Pi_{lh} = (1 - \frac{c_h}{\gamma a})\frac{\gamma a(a - c_l/\gamma)}{2} = \frac{(\gamma a - c_h)(a - c_l/\gamma)}{2} \tag{48}$$

(48) is smaller than (45) if the following holds:

$$\frac{2}{\gamma(1+\gamma)} < \frac{\gamma a - c_l}{\gamma a - c_h} \tag{49}$$

The left side of this inequality is decreasing in γ and the right side is increasing in γ. So we have two cases. *If* $\frac{a-c_l}{a-c_h} < 2$, the condition (49) does not hold for $0 < \gamma \leq 1$ and a separating equilibrium does not exist. Otherwise it holds if γ is sufficiently high.

Secondly, in order to have an equilibrium, h should not have an incentive to switch to reward-based crowdfunding. In this case, this is a trade-off between bankruptcy cost and the cost of moral hazard. If h switches to reward-based crowdfunding its payoff equals:

$$\Pi_{hl} = \frac{\gamma(1+\gamma)(a - c_h/\gamma)^2}{4}$$

This is less than (44).

Consider a situation where h selects reward-based crowdfunding and l selects profit-sharing.

Consider firm l. Similarly to the above analysis we have: $p_1 = p_2 = \frac{a+c_l/\gamma}{2}$, $\alpha = \frac{c_l}{\gamma a}$ and the entrepreneur‘s expected profit over the two periods equals:

$$\frac{(\gamma a - c_l)(a - c_l/\gamma)}{2} \tag{50}$$

Consider firm h. We have $p_1 = p_2 = \frac{a+c_h/\gamma}{2}$.

The firm's profit over the two periods equals

$$\Pi_h = \frac{\gamma(1+\gamma)(a - c_h/\gamma)^2}{4} \tag{51}$$

Suppose that h mimics l and chooses equity-based crowdfunding instead. h's profit Π_{hl} then equals

$$\Pi_{hl} = (1 - \alpha_l)(\gamma p_{1h}(a - p_{1h}) - c_l(a - p_{1h}) + \gamma p_{2h}(a - p_{2h}) - c_l(a - p_{2h}))$$

It equals

$$\Pi_{hl} = \frac{(\gamma a - c_l)(a - c_h/\gamma)}{2}$$

This is greater than (51) because $c_l < c_h$ and therefore such an equilibrium does not exist.

Proof of Proposition 6.

Consider reward-based crowdfunding. In period 2, the firm chooses p_2 to maximize $(p_2 - c - b)(s_r a - p_2)$ which makes $p_2 = \frac{s_r a + b + c}{2}$ (all calculations are identical to section 2.1. except that the cost equals $c + b$).

In period 1, the firm maximizes $(p_1 - b - c)(a - p_1) - I$ subject to $p_1 q_1 = (p_1 - c)(a - p_1) \geq I + cq_1 = I + c(a - p_1)$.

Two cases are possible. If

$$(a - c - b)^2 < 4I \tag{52}$$

then the firm will not be able to raise enough funds to launch the production. Otherwise we have $p_1 = \frac{a+c+b}{2}$.

The firm's profit over the two periods equals

$$\Pi_r = \frac{(a - b - c)^2}{4} + \frac{(s_r a - b - c)^2}{4} - I \tag{53}$$

Consider equity-based crowdfunding. In period 2, the firm chooses p_2 to maximize $(1 - \alpha)(p_2 - c - b)(s_e a - p_2)$ which makes $p_2 = \frac{s_e a + b + c}{2}$.

In period 1, the firm maximizes $(1 - \alpha)(p_1 - b - c)(a - p_1)$ which makes $p_1 = \frac{a+b+c}{2}$.

The firm's profit equals

$$\Pi_e = (1 - \alpha)(\frac{(a - b - c)^2}{4} + \frac{(s_e a - b - c)^2}{4})$$

Since

$$\alpha(\frac{(a - b - c)^2}{4} + \frac{(s_e a - b - c)^2}{4}) = I$$

we have:

$$\Pi_e = \frac{(a - b - c)^2}{4} + \frac{(s_e a - b - c)^2}{4} - I \tag{54}$$

In the case of bank loan financing we have $p_1 = p_2 = \frac{a+c}{2}$.

The firm's profit is:

$$\Pi_b = \frac{(a - c)^2}{2} - I \tag{55}$$

Since (53) is greater than (54) we have two cases. If I is sufficiently small ($(a - c - b)^2 \geq 4I$), resulting from the comparison of (53) and (54), the firm prefers reward-based crowdfunding to equity-based crowdfunding because $s_r > s_e$. As follows from the comparison of (53) and (55), the firm selects reward-based crowdfunding if s_r is sufficiently large or b is sufficiently small. This is not surprising given that b reflects the degree of the moral hazard cost under crowdfunding and s_r reflects the efficiency of market feedback. Otherwise, the firm takes a bank loan.

Let us now analyze the role of demand (a) on a firm's decision-making. The firm is indifferent between reward-based crowdfunding and a bank loan if:

$$\frac{(a - b - c)^2}{4} + \frac{(s_r a - b - c)^2}{4} - I = \frac{(a - c)^2}{2} - I$$

This equation can be rewritten as:

$$\frac{(a-b-c)^2}{2}+\frac{(s_r a-b-c)^2}{2}=(a-c)^2 \tag{56}$$

Since this is a quadratic equation, it implies that for any given value of I, the firm selects equity-based crowdfunding if a is either very small or very large. Otherwise it takes a bank loan.

If I is sufficiently large, the firm will not be able to use reward-based crowdfunding. As follows from the comparison of (54) and (55), the firm selects equity-based crowdfunding if s_e is sufficiently large or b is sufficiently small. Otherwise, the firm takes a bank loan.

The firm is indifferent between equity-based crowdfunding and a bank loan if:

$$\frac{(a-b-c)^2}{4}+\frac{(s_e a-b-c)^2}{4}-I=\frac{(a-c)^2}{2}-I$$

This equation can be rewritten as:

$$\frac{(a-b-c)^2}{2}+\frac{(s_e a-b-c)^2}{2}=(a-c)^2$$

then proceed in a similar fashion to the above analysis.

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